9.1 Power series, infinite series

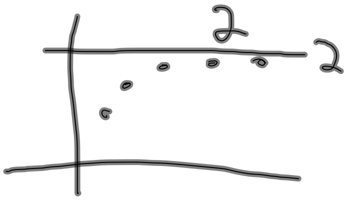
$$\sum_{n=1}^{\infty} \frac{1}{2^{n-1}} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \cdots = 2$$

Sequence of partial sums: converges to 2

n	s_n
1	1
2	$1+\frac{1}{2} = 1.5$
3	$1+\frac{1}{2}+\frac{1}{4} = 1.75$
4	$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8} = 1.8125$

$$u1(n) = \sum_{k=1}^{n} \frac{1}{2^{k-1}}$$

$\downarrow$ 2



Feb 10-1:30 PM

$$\boxed{\begin{array}{l}\text{geometric series} \\ a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} \cdots\end{array}}$$

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \cdots = \frac{1}{1-\frac{1}{2}} = 2$$

$a = 1 \quad r = \frac{1}{2}$

$$\boxed{\text{If } |r| < 1, \quad \sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}}$$

Feb 10-1:43 PM

$$2-\frac{4}{3}+\frac{8}{9}-\frac{16}{27}+\cdots \stackrel{?}{=} \frac{2}{1-\frac{-2}{3}}=\frac{2}{\frac{5}{3}}=\frac{6}{5}$$

geo? yes $a=2$, $r=-\frac{2}{3}$

$$\sum_{n=1}^{\infty} 3\left(\frac{5}{4}\right)^{n-1} \stackrel{?}{=} \infty \text{ diverges}$$

$$\boxed{\text{if } |r|\geq 1,\ \sum_{n=1}^{\infty} a r^{n-1} \text{ diverges}}$$

Feb 10-1:46 PM

new way to define a function if $|x|<1$

$$1+x+x^2+x^3+x^4+\cdots = \frac{1}{1-x}$$

geo? yes $a=1$ $r=x$

power series

$|x|<1$

$-1<x<1$

interval of convergence

Feb 10-1:55 PM

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - \cdots$$

$a = 1$

$r = -x$

integrate both sides

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} \cdots$$

$x = 1$ $\qquad \ln(2) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \cdots$

read ex 4 to see how to diff. a series

Feb 10-2:03 PM